

Sampling Methods

- Examine ^{first} 50 pages in 20 most recent issues,
 - Convenience sample
- Put the issues in a random order and examine every 10th page until 1000 have been examined
 - Systematic sample
- Assign a unique number to each page, use a computer to randomly select 1000 of those #'s
 - Simple random sample
- Randomly select 4 issues, examine every page in those issues
 - Cluster random sample (all pages from some)
- Examine 20 randomly selected pages from each of the 50 issues
 - Stratified random sample (some from all)

Experimental
Design

- Randomized Block
 - Subjects divided into groups on some other variable (like gender) before being randomly assigned or treatments are randomly assigned to each block

Experimental
Design

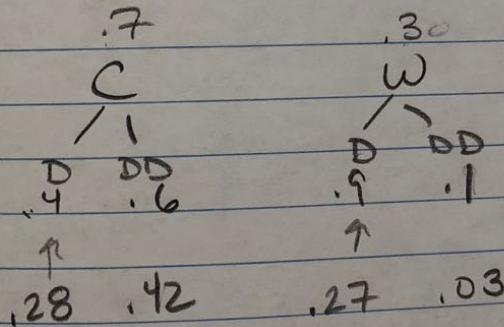
- Matched pairs
 - subjects are in both exp. + control groups or put into pairs w/ another subject

Probability Rules

Target Ex. 0.95 $1 - (0.95)^{10}$

1 2 3 4 5 6 7 8 9 10
.05

Gate Ex $\frac{30}{100} \cdot \frac{9}{10} = \frac{27}{100}$



Expected Value

- ① X is a discrete random variable. The table below defines a probability distribution for X .

What is the expected value of X ?

x	$P(X=x)$
-40	0.12
-30	0.04
-20	0.05
-10	0.17
0	0.62

- Step 1:
Multiply each value by its probability

x	$P(X=x)$	$x \cdot P(X=x)$
-40	0.12	-4.8
-30	0.04	-1.2
-20	0.05	-1
-10	0.17	-1.7
0	0.62	0

- Step 2:
Add results from step 1:

$$(-4.8) + (-1.2) + (-1) + (-1.7) + (0) = -8.7$$

Expected
Value
(EV)

② Coffee cups

+12% chance of getting fined

$$10,000 - 10,000 = -90,000$$

↑ ↑
Savings of - fine
cups

x	x	$P(X=x)$	$x \cdot P(X=x)$
Fined	-90,000	.12	-10,800
Not fined	10,000	.88	8,800
			<u>-2000 EV</u>

③ Sum of 2 fair 6-sided dice is 7

2	3	4	5	6	7
3	4	5	6	7	8
4	5	6	7	8	9
5	6	7	8	9	10
6	7	8	9	10	11
7	8	9	10	11	12

x	$P(X=x)$	$x \cdot P(X=x)$
-200	.25	-50
-100	.25	-25
0	.25	0
100	.25	25
		<u>-50 EV</u>

⑤ Cell Phone Insurance

$$\text{Breaks} = -300 + 800 = 500$$

x	$x(\text{cost})$	$P(X=x)$	$x \cdot P(X=x)$
Breaks	500	.20	100
Doesn't Break	-300	.80	-240
			<u>-140 EV</u>

⑥ Clothing Promotion

$$\text{Discount} = 300 \cdot .60 = 180$$

	x	$P(X=x)$	$x \cdot P(X=x)$
Heads	180	.50	90
Tails	300	.50	150
			<u>240 EV</u>

⑦ Poker Tournament

$$\text{Win} = -10,000 + 500,000 = 490,000$$

	x	$P(X=x)$	$x \cdot P(X=x)$
Win	490,000	.005	2450
Lose	-10,000	.995	-9950
			<u>-7500 EV</u>

⑧ Train in France

$$\text{Fine} = 5 + 300 = 305$$

	x	$P(X=x)$	$x \cdot P(X=x)$
Fine	305	.05	15.25
No Fine	5	.95	4.75
			<u>20 EV</u>

⑨

x	$P(X=x)$	$x \cdot P(X=x)$
0	.35	0
10	.25	2.5
20	.4	8
		<u>10.5 EV</u>

⑩ Cellphone Insurance #2
 Breaks = $-200 + 1000 = 800$

	x	$P(X=x)$	$x \cdot P(X=x)$
Breaks	800	.10	80
Doesn't Break	-200	.90	-180
			<u>-100 EV</u>

⑪

x	$P(X=x)$	$x \cdot P(X=x)$
-7	0.2	-1.4
-3	0.1	-.3
3	0.4	1.2
7	0.3	2.1
		<u>1.6 EV</u>

Standard Deviation of a Discrete Random Variable

- Flood Insurance

	No Flood	Flood
$X = \text{profit}$	\$200	-\$99,800
$P(X)$	0.999	0.001

$\mu_x = \$100$ Calculate σ_x

$$\text{Var}(X) = \sigma_x^2$$

$$= (x_1 - \mu_x)^2 p_1 + (x_2 - \mu_x)^2 p_2 + \dots +$$

$$= \sum (x_i - \mu_x)^2 p_i$$

$$\sigma_x = \sqrt{\text{Var}(X)}$$

$$\sigma_x = \sqrt{(200 - 100)^2 0.999 + (-99,800 - 100)^2 0.001}$$

- Jenna goes fishing

$$\sqrt{(0 - 1.6)^2 0.04 + (1 - 1.6)^2 0.32 + (2 - 1.6)^2 0.64}$$

- Gloria enjoys Skee Ball

$$\sqrt{(100 - 19)^2 .10 + (10 - 19)^2 .90}$$

- Talia wants to play a basketball game

$$\sqrt{(-5+1)^2 \cdot 36 + (0+1)^2 \cdot 48 + (5+1)^2 \cdot 16}$$

- Renter's Insurance

$$\sqrt{(60-50)^2 \cdot 999 + (-9940-50)^2 \cdot 001}$$

Probability in Normal Curves

- use normalcdf in calculator

[2nd] [Vars] normalcdf [lower, upper, μ , σ]

Transforming Random Variables

- Mr. Gupta's 3 question Quiz

$$Y = 10x + 5$$

$$\begin{array}{l|l} \mu_y = 10(\mu_x) + 5 & \sigma_y = 10(\sigma_x) \\ 10(1.95) + 5 & = 10(.8) \\ 19.50 + 5 & = 8 \\ 24.50 & \end{array}$$

- Nikola plays horseshoes

$$\begin{array}{l|l} \mu_y = 3(\mu_x) & \sigma_y = 3(\sigma_x) \\ 3(.3) & 3(.56) \\ .9 & \end{array}$$

* Must say independent

Combining Random Variables

- Factory w/ 3-step process $\mu=10$ $\sigma=5$

- Calculate variance for each step

$$\begin{aligned}\sigma_{x+y+z}^2 &= \sigma_x^2 + \sigma_y^2 + \sigma_z^2 \\ &= 5^2 + 5^2 + 5^2 \\ &= 25 + 25 + 25 \\ &= 75\end{aligned}$$

- S.D. = $\sqrt{\text{Variance}} = \sqrt{75}$

- Find mean of T

$$\mu_L + \mu_C = \mu_T$$

- Calculate SD of 4 cookies

$$V = (0.25)^2 + (0.25)^2 + (0.25)^2 + (0.25)^2$$

$$V = .25$$

$$\sigma = \sqrt{.25} = .5$$

- Andy + Jim
Find SD of D ($D = A - J$)

$$V = 1^2 + .5^2$$

$$= 1.25$$

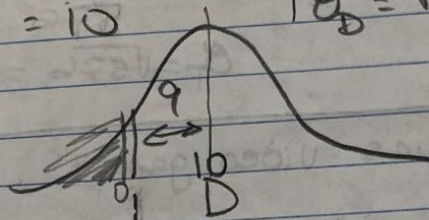
$$SD(\sigma) = \sqrt{1.25} =$$

Combining Normal Random Variables

- Heights of men & women

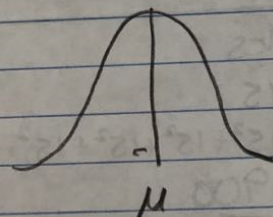
$$\begin{aligned} \textcircled{1} D &= M - W \\ \mu_D &= \mu_M - \mu_W \\ &= 172 - 162 \\ &= 10 \end{aligned}$$

$$\begin{aligned} \textcircled{2} \sigma_D^2 &= \sigma_M^2 + \sigma_W^2 \quad (\text{Variance}) \\ &= (7.2)^2 + (5.4)^2 \\ &= 81 \\ \sigma_D &= \sqrt{81} = 9 \quad (\text{SD}) \end{aligned}$$



$$\textcircled{3} \text{Normalcdf}(\text{lower}, \text{upper}, \mu, \sigma) = \text{Normalcdf}(-99999, 0, 10, 9)$$

- Company produces candles + access



	μ	σ
Candle	500	15
stand	200	8
	700	17

$$\text{normalcdf}(-9999, 683, 700, 17)$$

- Notions & exams 2 students

$$\begin{aligned} \mu &= 41 \quad \sigma = 9 \\ D &= X_1 - X_2 \\ &= 41 - 41 \\ &= 0 \\ \sigma^2 &= 9^2 + 9^2 \\ &= 162 \\ \sigma &= \sqrt{162} \\ \text{normalcdf} &(-10, 10, 0, \sqrt{162}) \\ &= 2 \times (15, 9999, 0, \sqrt{162}) \end{aligned}$$

Cont

Combining Normal Random Variables

- Breakfast cereal

$\mu_F = 370$

$\sigma_F = 24$

$\mu_R = 170$

$\sigma_R = 7$

$\mu = 370 + 170$
540

$\sigma^2 = 24^2 + 7^2$

$\sigma^2 = 625$

$\sigma = 25$

Ali faster

- Ali + Bruno run 400m dash

$\mu_A = 59$

$\sigma_A = 4.8$

$\mu_B = 50$

$\sigma_B = 3.6$

$A - B = 59 - 50$

9

$\sigma^2 = 4.8^2 + 3.6^2$

$\sigma^2 = 36$

$\sigma = 6$

Normalcdf(-99999, 0, 9, 6)

- Car manuf. Value + seal

$\mu_V = 50$

$\sigma_V = .3$

$\mu_S = 51$

$\sigma_S = .4$

$S - V = 51 - 50$

-1 1 3

$\sigma^2 = .3^2 + .4^2$

$\sigma^2 = .25$

$\sigma = .5$

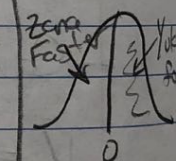
ncdf(0, 2, 1, 0.5)

Yuki + Zeno

$\mu_Y = 80$

$\mu_Z = 85$

$Y - Z = 5$



Carnival ride for 4 passengers

$$\mu_W = 65 \quad \sigma = 12$$

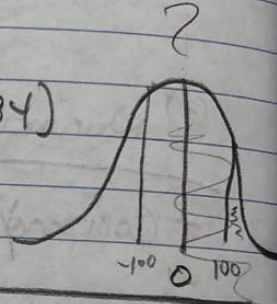
$$\begin{aligned} \mu_T &= \mu_W^1 + \mu_W^2 + \mu_W^3 + \mu_W^4 \\ &= 65 + 65 + 65 + 65 \\ &= 260 \end{aligned}$$

$$\begin{aligned} \sigma_T^2 &= \sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_4^2 \\ &= 12^2 + 12^2 + 12^2 + 12^2 \\ &= 576 \\ \sigma_T &= \sqrt{576} = 24 \end{aligned}$$

Angelita + Cynthia's video games

$$\begin{aligned} D &= A - C \\ 900 - 800 \\ \mu &= 100 \\ \text{A wins} \\ \sigma^2 &= 96^2 + 72^2 \\ &= (9216 + 5184) \\ \sigma &= \sqrt{14400} \\ \sigma &= 120 \end{aligned}$$

Normalcdf(0, 99999, 100, 120)



Company produces apples

$$\begin{aligned} \mu &= 175 \\ \mu_W &= 175 \times 4 \\ &= 700 \\ \sigma &= 15 \\ \sigma_T^2 &= 15^2 + 15^2 + 15^2 + 15^2 \\ &= 900 \\ \sigma &= \sqrt{900} = 30 \end{aligned}$$

Sam + Taylor's car wash

$$\begin{aligned} D &= S - T \\ 20 - 18 \\ 2 \\ -10 \quad 10 \\ \sigma_S &= 6.4 \quad \sigma_T = 4.8 \\ \sigma_T^2 &= 6.4^2 + 4.8^2 \\ \sigma_T^2 &= 64 \\ \sigma_T &= \sqrt{64} = 8 \end{aligned}$$

Binomial Variable - coin flip

- Made up of independent trials
 - flipping coins is ind.
 - picking ppl is not, unless sample size is less than 10% of population
- each trial can be classified as a success or failure
- fixed # of trials
- probability of success on each trial is constant

Not Binomial - pulling cards from deck w/o replacement

Binomial Probability Formula

- Online retailer, next-day shipping
- 95% get package - get 20 orders SRS

$$P(X=19) = (0.95)(0.95) \dots (0.95)(0.05) \\ (0.95)^{19}(0.05)$$

Binomial coefficient $\binom{20}{19}$ ← arrangements
 ← successes

-
- Heather's 60% Heads coin
 - flip 5 times
 - get exactly 3 heads

$$\binom{5}{3} (.6)^3 (.4)^2$$

Calculating binomial probability

- Aja's favorite cereal

- 1 in 4 contain prize

- buys 5 boxes

← cdf (less than or =)

$P(X \leq 1)$ wins at most 1 prize

calc binomcdf (5, 0.25, 1)
trials x value
prob

- Heather's coin 60% heads

← exactly 3 (binompdf)
 $P(X=3)$ 5 flips

binompdf (5, .60, 3)

- Marta Basketball 90% make

← greater than or =

$P(X \geq 2)$ 3 shots

$P(X \leq 1)$ binomcdf (3, .9, 1) = 0.028

$P(X \geq 2) = 1 - P(X \leq 1)$

= 1 - 0.028

= 0.972

- Layla's coin 60% 5 flips

$P(X > 3)$

* if $> \#$, $P(X < \#)$

$P(X < 3) = 1 - P(X \leq 3)$

Mean and Standard Deviation of a Binomial Random Variable

—Candy machine

$p =$
1,000 pieces 30% blue

$n =$
SRS of 15 pieces $X = \#$ of blue

Find $\mu_x + \sigma_x$

$$\mu_x = np$$

$$\begin{aligned}\mu_x &= 15(.3) \\ &= 4.5\end{aligned}$$

$$\sigma_x = \sqrt{np(1-p)}$$

$$= \sqrt{15(.3)(1-.30)}$$

$$= \sqrt{3.15}$$

$$= 1.775$$

Geometric Random Variables

- Not a fixed # of trials
- How many trials until success?

Finding Probability of GRV

- Jeremiah 3-point shot 25% makes shot

$$P(M=3^{\text{rd}} \text{ attempt}) = \overset{\text{miss}}{\frac{3}{4}} \cdot \overset{\text{miss}}{\frac{3}{4}} \cdot \overset{\text{make}}{\frac{1}{4}} = \frac{9}{64}$$

Cumulative Geometric Probability (greater than value)

- Emelia @ DMV SUV's = 12%
V = # of vehicles she registers in a day
until she registers an SUV

$P(E \text{ registers more than 4 before an SUV})$

$$P(V > 4) = P(V=5) + P(V=6) + P(V=7) + \dots$$

$$\begin{array}{c} \downarrow \\ P(V \text{ not } \leq 4) = P(\text{first 4 cars not SUV}) = (1-.12)^4 \\ = 0.5997 \end{array}$$

Cumulative Geometric Probability (less than a value)

- Lilyana cake decorating 10% orders over phone

Let C = # of cake orders until a phone order

$$\begin{aligned}\text{Find } P(C < 5) &= P(C=1) + P(C=2) + P(C=3) + P(C=4) \\ &= (.1) + (.9 \cdot .1) + (.9^2 \cdot .1) + (.9^3 \cdot .1) \\ &= 1 - P(\text{No telephone in 4 orders}) = 1 - (.9)^4\end{aligned}$$

Calculator Methods

- Keep picking cards until a ~~king~~^{king}, replace if not ^{king}
 $\leftarrow P(\text{king})$

$$- P(\text{pick 5}) = \text{geometpdf}\left(\frac{1}{13}, 5\right) = 0.0558$$

$$- P(X \leq 10) = P(X \geq 9) = P(X=1) + \dots + P(X=9) \\ \text{geometcdf}\left(\frac{1}{13}, 9\right) = 0.5134$$

$$- P(X > 12) = 1 - P(X \leq 12) \\ 1 - \text{geometcdf}\left(\frac{1}{13}, 12\right) = 0.3827$$

Calculating Percentiles

- percentile: % of the data that is below the amt in question

: % of the data that is at or below the amt in quest.

Expected Value with Empirical Probabilities

① Birthday Die

<u>Die Value</u>	<u>Freq.</u>	<u>Rel. Freq.</u>	<u>V. Rel Freq.</u>
1	90	$\frac{90}{500} = .18$	.18
2	110	$\frac{110}{500} = .22$	.44
3	95	$\frac{95}{500} = .19$	.57
4	70	$\frac{70}{500} = .14$	.56
5	75	$\frac{75}{500} = .15$	.75
6	60	$\frac{60}{500} = .12$	.72
	<u>500 total</u>		<u>3.22 EV</u>

$$20 \cdot 3.22 = 64.4 \quad \frac{20 \cdot 110}{60} = 1.83$$

② Frisbee

<u>Distance</u>	<u>Throws</u>	<u>Rel. Freq.</u>	<u>D. RF</u>
10	4	.04	.4
20	21	.21	4.2
30	32	.32	9.6
40	35	.35	14
50	8	.08	4
	<u>100</u>		<u>32.2 EV</u>

$$50 \cdot 32.2 = 1610 \quad \frac{40 \cdot 35}{8} = 4.4$$

③ Salesman Sam

<u># of Clients (x)</u>	<u>Days</u>	<u>RF</u>	<u>x · RF</u>
1	153	.3825	.3825
2	110	.275	.55
3	77	.1925	.5775
4	39	.0975	.39
5	21	.0525	.2625
	400 tot		2.1625 EV

$$5 \cdot 2.1625 = 10.8 \approx 11 \quad \begin{matrix} \textcircled{2} = 110 \\ \textcircled{3} = 21 = 5.23 \end{matrix}$$

④ Divya

<u>Pieces in Set</u>	<u>Sets</u>	<u>Tot Pieces</u>
3	x	$3x$
5	5	25
10	2	20

$$\begin{array}{c|c} 45+3x & \\ \hline 7+x & 5 \end{array}$$

$$5(7+x) = 45+3x$$

$$35+5x = 45+3x$$

$$\begin{array}{r} -3x \quad -35 \\ \hline 2x = 10 \end{array}$$

$$2x = 10$$

$$x = 5$$

⑤ Alma's Lawn Care

Acres	Customers	A.C	
1	4	4	- 4
2	6	12	-12
3	x	$3x$	-45
4	y	$4y$	114
5	9	45	
	50	175	

$$x + y = 31$$

$$y = 31 - x$$

$$3x + 4y = 114$$

$$3x + 4(31 - x) = 114$$

$$3x + 124 - 4x = 114$$

$$-x + 124 = 114$$

$$-x = -10$$

$$-x = -10$$

$$x = 10$$

⑥ Rohit the Runner

Distance	Runs	RF	D.RF
1	6	.12	.12
2	13	.26	.52
3	31	.62	1.86
	50		2.5 EV

⑦ Daniela Tennis

<u>Sets Won</u>	<u>Matches</u>	<u>S.M</u>
0	$\boxed{x}$ 2	0
1	4	4
2	$\boxed{y}$ 4	$2y$
	10	12

$$2y = 8$$
$$y = 4$$

⑧ Easter Eggs

<u>Eggs Found</u>	<u>Hunts</u>	<u>E.H</u>
8	2	16
11	4	44
14	$\boxed{x}$	$14x$
	$6+x$	

$60+14x$

$6+x \mid 11.6$

$$60+14x = 11.6(6+x)$$
$$60+14x = 69.6+11.6x$$
$$2.4x \quad 9.6$$

⑨ Rohit's Races

Total = 3450

Length	Races	Rel. Freq	Rel Freq · Length
100	3	$\frac{3}{16} = .1875$	18.75
200	X	$\frac{x}{16}$	$\frac{200}{16}x$
400	2	$\frac{2}{16} = 0.125$	50
800	Y	$\frac{y}{16}$	$\frac{800}{16}y$
	16	1.00	431.25

$$EV = \frac{3450}{8} = 431.25$$

$$3 + x + 2 + y = 16$$

$$x + y + 5 = 16$$

$$x + y = 11$$

$$18.75 + \frac{200}{16}x + 50 + \frac{800}{16}y = 431.25$$

$$\frac{200}{16}x + \frac{800}{16}y + 68.75 = 431.25$$

$$16 \left(\frac{200x}{16} + \frac{800y}{16} \right) = 362.5$$

$$200x + 800y = 5800$$

$$- 200x - 200y = -2200$$

$$\hline 600y = 3600$$

$$600 \quad 600$$

$$y = 6$$

$$x = 5$$

⑩ Hector's Paintings

Area	Survey Resp	A.S
10	3	30
20	5	100
30	11	330
40	11	440
Tot	30	900

$900/30 = 30$

⑪ Chef Jeff

Chick	Orders	
3	30 $\boxed{x}$	3x
6	20	120
8	50 $\boxed{y}$	8y
Tot	100	610

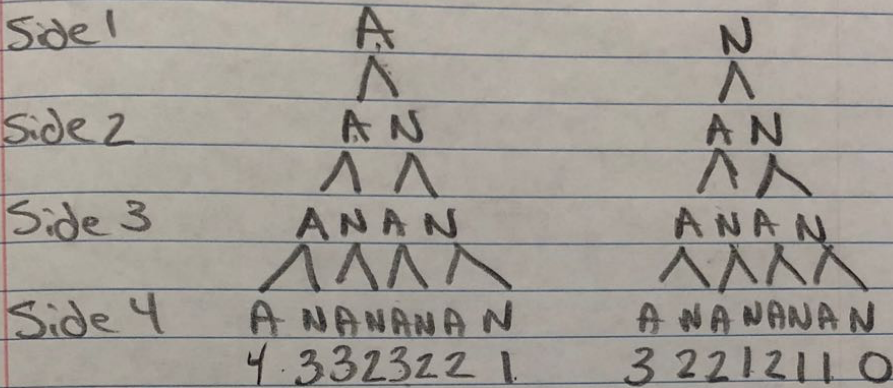
$610/100 = 6.1$

$$\begin{aligned} x + 20 + y &= 100 & 3x + 120 + 8y &= 610 \\ x + y &= 80 & 3x + 8y &= 490 \end{aligned}$$

$$\begin{aligned} 3x + 8y &= 490 \\ -3(x + y) &= -80 \\ \hline 5y &= 250 \\ y &= 50 \end{aligned}$$

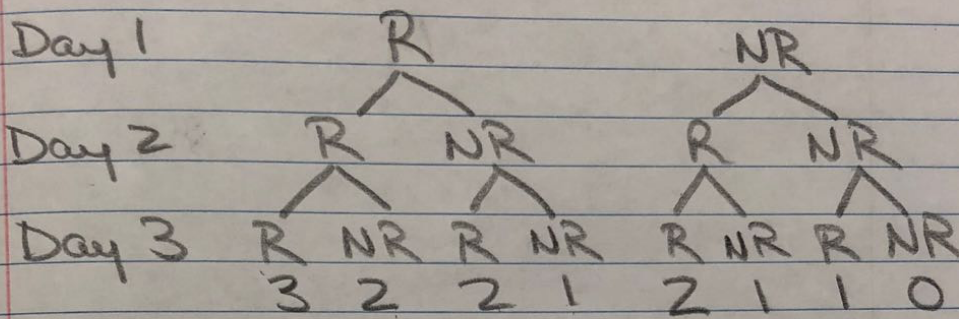
Expected Value w/ Calculated Probs.

① Artifacts



Artif			Value	
0	1	$\frac{1}{16}$	0	0
1	4	$\frac{4}{16}$	100	25
2	6	$\frac{6}{16}$	250	93.75
3	4	$\frac{4}{16}$	500	125
4	1	$\frac{1}{16}$	1000	62.50
				306.25 ✓

② Car Dealership Rain



Rainy Days			Revenue	
0	1	$\frac{1}{8}$	3000	= 375
1	3	$\frac{3}{8}$	1800	= 675
2	3	$\frac{3}{8}$	1300	= 487.5
3	1	$\frac{1}{8}$	800	= 100
			1637.50	

③ Bob & Anna Lunch

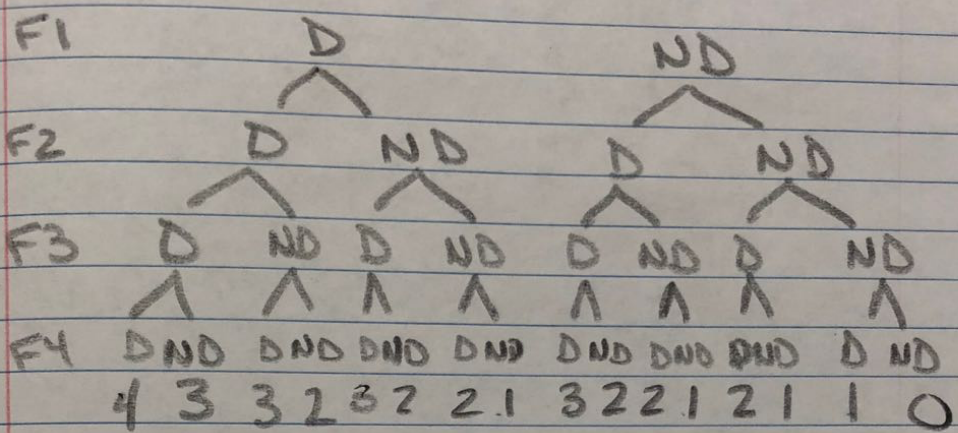
①, 1	2, 1	3, 1	4, 1
1, 2	②, 2	3, 2	4, 2
1, 3	2, 3	③, 3	4, 3
1, 4	2, 4	3, 4	④, 4

Meet $\frac{4}{16} \cdot 5 = 1.25$

Don't Meet $\frac{12}{16} \cdot 10 = 7.50$

8.75

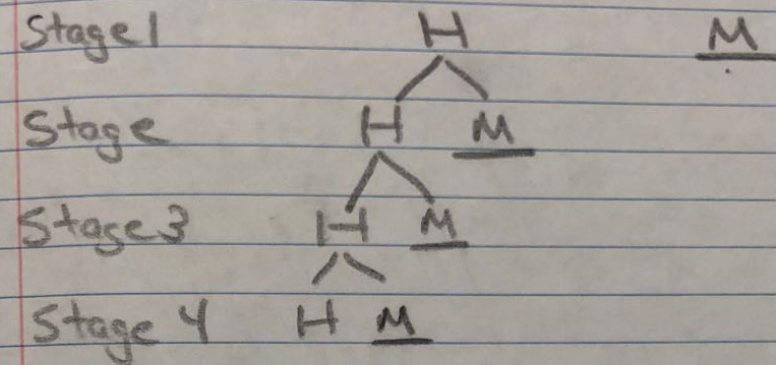
④ Fireworks



<u>Duds</u>			<u>Value</u>	
0	1	$\frac{1}{16}$	1000	62.50
1	4	$\frac{4}{16}$	700	175.00
2	6	$\frac{6}{16}$	500	187.50
3	4	$\frac{4}{16}$	100	25.00
4	1	$\frac{1}{16}$	0	<u>0</u>

65

⑤ Rory the Irish Gunman



Stage	Prob.	Amt	
1	.6	0	0
2	.6 : .5 = .3	100	30
3	.3 : .4 = .12	300	36
4	.12 : .3 = 0.36	1000	<u>36</u>

⑥ Ukad + Ulga Roll Dice

2	③	4	5	⑥	7
③	4	5	⑥	7	8
4	5	⑥	7	8	⑨
5	⑥	7	8	⑨	10
⑥	7	8	⑨	10	11
7	8	⑨	10	11	⑫

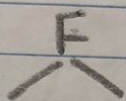
$$\text{Div by 3: } \frac{12}{36} = \frac{1}{3} \cdot 6 = 2$$

$$\text{Not Div ; } \frac{24}{36} = \frac{2}{3} \cdot -3 = -2$$

By 3

⑦ Clumsy Carl

Spot 1:



N

Spot 2:

F

N

F

N

Spot 3:

F

N

F

N

F

N

F

N

3

2

2

1

2

1

1

0

Falls

#PPI

0 (1/8)

0

= 110.0

1 (3/8)

500

= 187.5

2 (4/8)

800

= 300.0

3 (1/8)

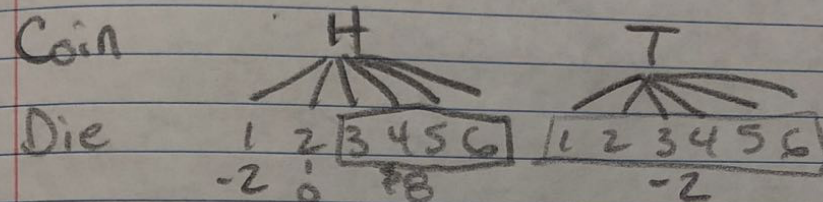
900

= 112.5

600.0

⑧ Jorge @ Casino

Flip Coin & Roll 1 die



Prize

-2 · $\frac{1}{12}$	-1.17
0 · $\frac{1}{12}$	0
8 · $\frac{4}{12}$	<u>2.67</u>

⑨ Poja & Amit Card Game

Outcome

Face $(\frac{3}{13}) \cdot 6 = 1.38$

Ace $(\frac{1}{13}) \cdot 4 = .31$

Other $(\frac{9}{13}) \cdot -2 = -1.38$

⑩ Cecilia + Layla Game
 R_1, R_2, R_3 $G_1, G_2, G_3, G_4, G_5, G_6, G_7$

Red R ① 2 ③

 Green ① ② ③ ④ ⑤ ⑥ ⑦ 1 2 3 4 5 6 7 ① ② ③ ④ ⑤ ⑥ ⑦

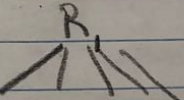
Outcome

O-O ($\frac{8}{21}$) \$10 3.81

O-E ($\frac{6}{21}$) 0 0

Other ($\frac{7}{21}$) -8 -2.67

Vera + Alexey - Markers

R_1 R_2 R_3 R_4 R_5

 G_1 G_2 8 9 10 6 7 8 9 10 6 7 8 9 10 6 7 8 9 10 6 7 8 9 10
 5 < 4 < 3 < 2 < 1 <
 1 = 1 = 1 = 1 = 1 =
 1 > 1 > 2 > 3 >

>12 $\frac{6}{25}$ \$5 1.2

=12 $\frac{4}{25}$ 0 0

<12 $\frac{15}{25}$ -6 $\frac{-3.6}{-2.4}$

Making Decisions w/ Expected Values

① Mohammed + His Startup Companies

Small Comp.

5@40%

\$10,000 if succ.

-1,000,000 if unsuc.

$$.40^5 = .01 \cdot 10 = .1$$

$$1 - .01 = .99 \cdot -1 = -.99$$

$$EV_{sm} = -.89$$

Large Comp.

3@30%

\$15m if succ.

-5m if unsucc.

$$.30^3 = .027 \cdot 15 = .405$$

$$1 - .027 = .973 \cdot -5 = -4.865$$

$$-4.46$$

② Samuel the Ent.

Small

4@90%

\$10m if suc.

-3m if unsuc.

$$.90^4 = .6561 \cdot 10 = 6.561$$

$$1 - .6561 = .3439 \cdot -3 = -1.0317$$

$$5.5m$$

Large

2@20%

20m if suc.

-2m if unsuc.

$$.2^2 = .04 \cdot 20 = .8$$

$$1 - .04 = .96 \cdot -2 = -1.92$$

$$-1m$$

③ You + Friend Coin game

Game 1

- $\frac{1}{16}$ - H H H H $\frac{9}{16} = \frac{1}{4} \cdot 5 = 1.56$
- H H H T $\frac{1}{4} \cdot -2 = -1.38$
- H H T H $.185$
- H H T T
- H T H H
- H T H T
- H T T H
- H T T T -
- T H H H
- T H H T
- T H T H
- T H T T -
- T T H H
- T T H T -
- T T T H -
- T T T T -

Game 2

$\frac{1}{64} \cdot 150 = 2.34$
 $\frac{63}{64} \cdot -2 = -1.97$
 $= .38$

$\leq 2 = \$1 \quad \frac{11}{16} = .6875$
 $> 2 = \$3 \quad \frac{5}{16} = .3125$
 $= +.25$

$\frac{1}{32} \cdot 100 = 3.125$
 $\frac{31}{32} \cdot -4 = -3.875$
 $= -.75$

④ Minneapolis

Car Result	Stoplight		Stop Sign
Major Acc	15	16.875	19
Minor Acc	24	18	31
No Acc	761	0	750
Total	800	34.88	800

4000 cars	75.	67,500	95.	66,500
	120	72,000	155	31,000

⑤ Chicago Intersection

$$\begin{array}{l} \text{Light} \\ 170 \cdot 600 = 102,000 \end{array}$$

$$\begin{array}{l} \text{Sign} \\ 220 \cdot 750 = 165,000 \end{array}$$

$$200 \cdot 400 = 80,000$$

$$270 \cdot 500 = 135,000$$

Diego Batting Cage

⑥

Slow Ball

$$\begin{array}{l} .95^{10} \\ \$25 \text{ if succ} \\ -5 \text{ if unsucc} \end{array}$$

$$\begin{array}{l} .95^{10} = 14,968 \\ = -2,006 \\ \hline 12,962 \sim 13 \end{array}$$

Fast Ball

$$\begin{array}{l} 3 @ .60 \\ \$60 \text{ if succ} \\ -10 \text{ if unsucc} \end{array}$$

$$\begin{array}{l} .60^3 = 0.216 \cdot 60 = 12.96 \\ .784 \cdot -10 = -7.84 \\ \hline 5.12 \\ \sim 5 \end{array}$$

⑦ Fruit stand

$$\begin{array}{r} 246 \cdot .75 = 184.50 \\ 40 \cdot .80 \quad 32 = \\ \hline 216.50 \end{array}$$

$$\begin{array}{r} 100 \cdot .95 = 95 \\ 152 \cdot .60 = 91.20 \\ \hline 186.20 \end{array}$$

⑧ Bus Tkts

$$\begin{array}{r} 8.35 \cdot 20 = 167 \\ 1.65 \cdot 5 = 8.25 \\ \hline 175.25 \end{array}$$

$$\begin{array}{r} 7.3 \cdot 40 = 292 \\ 2.7 \cdot 15 = 40.50 \\ \hline 332.50 \end{array}$$

$$\begin{array}{r} \textcircled{9} \quad 7.2 \cdot 50 = 360 \\ 2.8 \cdot 10 = 28 \\ \hline 388 \end{array}$$

$$\begin{array}{r} 6.5 \cdot 70 = 455 \\ 3.5 \cdot 30 = 105 \\ \hline 560 \end{array}$$

$$\begin{array}{r} \textcircled{10} \quad 61 \cdot 3 = 183 \\ 62.8 \cdot 3 = 188.40 \\ \hline 371.40 \end{array}$$

$$\begin{array}{r} 43.8 \cdot 4.00 = 175.20 \\ 100.2 \cdot 2.50 = 250.50 \\ \hline 425.70 \end{array}$$

$$\begin{array}{l} \textcircled{11} \quad 6 @ 60 \text{ mph} @ 90\% \\ \$50 \text{ if succ} \\ -50 \text{ if unsuc} \end{array}$$

$$\begin{array}{l} 3 @ 90 \text{ mph} @ 70\% \\ 40 \text{ if succ} \\ -10 \text{ if unsuc} \end{array}$$

$$\begin{array}{r} .90^6 \cdot 50 = 26.572 \\ .468 \cdot -50 = -23.427 \\ \hline 3.14 \end{array}$$

$$\begin{array}{r} .343 \cdot 40 = 13.72 \\ (.657 \cdot -10) = -6.57 \\ \hline 7.15 \end{array}$$

~ 3

$\textcircled{12} \text{ R.P.S}$

L = Paper
P = Rock

L = Rock
P = Sc. .30

⑬ Tate's Virus

<u>Virus</u>	<u>Purchased</u>	<u>No Purch</u>
Pikachu	$100 - 25 = 25$	$200 \cdot 25 = 50$
Trojan	$100 - 15 = 15$	$100 \cdot 15 = 15$
ADA	$400 \cdot 10 = 40 + 100 = 140$	$400 \cdot 10 = 40$
	9140	105
	$(30+25) = 55$	(150)

⑭ Dina's Ice Cream

<u>Choc.</u>	<u>Vanilla</u>
$(61/100 \cdot 500) = 305 \times 2 = 610$	$.16 \cdot 500 = 80 \times 3 = 240$
$(12/100 \cdot 500) = 60 \times 2 = 120$	$.73 \cdot 500 = 365 \times 1 = 365$
730	605

⑮ Marvin's Moped

	<u>Light</u>	<u>No Light</u>
Morn	$2000 \cdot 10 = 200$	$2000 \cdot 10 = 200$
Dusk	$4000 \cdot 15 = 600$	$4000 \cdot 15 = 600$
Night	$—$	$2000 \cdot 20 = 400$
	$800 +$	1200
	150	
	950	

(16) Ming Fishing

	Large Lure	Small Lure
Tr	$330 \times 4 = 1320$	$90 \times 2 = 180$
SF	$80 \times 2 = 160$	$650 \times .5 = 325$
	<u>1480</u>	<u>505</u>

(17) Deniz RPS

	<u>Rock</u>			<u>Paper</u>
Rock	.50	—		.50 1000 = 500
Paper	.20	-1000 = -200		.20 —
Sciss	.30	+1000 = 300		.30 - 1000 = -300
		<u>-200</u>		<u>200</u>

(18) Francisco Arcade

3@30% \$50	4@80%	27.8528
$.30^3 \cdot 48 = 1.296$		<u>-1.1808</u>
$= -1.946$		26.672

(19) Felipe the Fisherman

	Large Lure	Small Lure
NP	$60 \cdot 5 = 300$	$66 \times 3 = 198$
W	$72 \cdot 3 = \frac{216}{516}$	$162 \times 1 = \frac{162}{360}$

(20) Jeremy Health Ins.

	Low Ded.	High Ded.
0	0	0
700	100	140
3000	75	150
5000	75	150
10,000	<u>50</u>	<u>100</u>
	6300	5540

(21) Sam Squirrel Monkey

Paint	Siding	
Red	Red	18 total
	Green	- 3 same
	White	15 Not same
Green	R	$\frac{3}{18} \cdot 1000 = 166.67$
	G	$\frac{3}{18} \cdot -100 = -83.33$
	W	<u>83.33</u>
W	R	
	G	
	W	

(22) Omar's Fruit Shop

Apples

$$\begin{array}{r} 302 \cdot .25 = 75.50 \\ 45 \cdot .45 = 20.25 \\ \hline 95.75 \end{array}$$

Bananas

$$\begin{array}{r} 46 \cdot .40 = 18.40 \\ 310 \cdot .30 = 93.00 \\ \hline 111.40 \end{array}$$