Independent and Dependent Events

Probability – $\frac{\#\text{ of possibilities that meet condition}}{\#\text{ of equally likely possibilities}}$

Coins: heads or tails only; 2 outcomes

Probability of heads? $\frac{1 \text{ heads}}{2 \text{ total outcomes}}$

- Probability of Rolling Dice

$$\frac{\#\text{ of outcomes that meet condition}}{\#\text{ of total outcomes}}$$

- Simple Probability

$$\frac{\#\text{ of outcomes that meet condition}}{\#\text{ of total outcomes}}$$

- Describing subsets of Sample Spaces
  - Read carefully

* Venn Diagrams and Adding Probabilities

- Adding Probabilities (OR)

OR = ADD

* Do not add same event twice

Independent and Dependent Events

- Independent Probability
  Calculate each probability and Multiply

- Probabilities of Compound Events
  - Create Sample Space
  - Count events

Dependent Probability

- Identifying Dependent and Independent Events
  P(A|B) probability of A occuring after B has occured

Ex. A = king B = 3

Events are independent if:
$P(A \text{ and } B) = P(A) \cdot P(B)$

× - $P(A|B) = P(A)$ $0 \neq 4/52$
× - $P(B|A) = P(B)$ $0 \neq 4/52$
× - $P(A \text{ and } B) = P(A) \cdot P(B)$ $0 = 4/52 \cdot 4/52 \Rightarrow 0 \neq 1/169$

- Multiplying Dependent Probabilities

Ex. Pair of 6-sided dice

A = 2 on 1st die — $1/6$

B = sum of dice is 7 or greater — $21/36 = 7/12$

x - Events are independent] same

x - $P(A \text{ and } B) = P(A) \cdot P(B)$ — $2/36 = 1/6 \cdot 7/12 \Rightarrow 1/36 \neq 7/72$

✓ - $P(A \text{ and } B) = P(A) \cdot P(B|A)$ — $2/36 = 1/6 \cdot 2/6 \Rightarrow 2/36 = 2/36$

✓ - $P(A \text{ and } B) = P(B) \cdot P(A|B)$ — $2/36 = 7/12 \cdot 2/21 \Rightarrow 1/18 = 1/18$

✓ - $P(A \text{ and } B) = 1/18$ — $2/36 = 1/18$

Ex. Hat contains 3 hearts + 4 spades

A = heart first — $P(A) = 3/7$

B = spade second — $P(B) = 4/7$

$P(A \text{ and } B)$ — $3/7 \cdot 4/6 = 4/14 = 2/7$

x - Events are independent] same

x - $P(A \text{ and } B) = P(A) \cdot P(B)$

✓ - $P(A \text{ and } B) = P(A) \cdot P(B|A)$ ALWAS TRUE

$3/7 \cdot 4/6 = 2/7$

$2/7 = 2/7$

x - $P(A \text{ and } B) = P(B) \cdot P(A|B)$ ALWAYS TRUE

$2/7 = 4/7 \cdot$ { heart → spade $= 3 \cdot 4 = 12$ outcomes

spade → spade $= 4 \cdot 3 = 12$ outcomes

24 total outcomes w/ spade 2nd

$7 \cdot 6 =$ total outcomes $= 42$

$P(B) = 24/42 = 4/7$

$P(A|B) = 1/2$

Ex: Hat contains 6 diamonds and 8 clubs

A: club first
$P(A) = 8/14 = 4/7$

B: diamond second
$P(B) = 3/7$

✓ – Independent ✓ Replaces card club → diamond $8 \cdot 6 = 48$
diamond → diamond $6 \cdot 5 = 30$
78
all outcomes $14 \cdot 13 = 182$

✓ – $P(A \text{ and } B) = P(A) \cdot P(B)$
$12/49 \quad 4/7 \cdot 3/7$

✓ – $P(A \text{ and } B) = P(B) \cdot P(A|B)$
$12/49 = 3/7 \cdot 4/7$ ✓

$P(A \text{ and } B) = P(A) \cdot P(B|A)$
$= 4/7 \cdot 8/13$

Ex: Basketball, soccer, football, hocky, + quidditch
Q = quidditch H: hockey
$P(Q) = 0.9 \quad P(H) = 0.6 \quad P(Q \text{ and } H) = 0.5$

Find $P(Q|H)$

$P(Q \text{ and } H) = P(H) \cdot P(Q|H)$
$\frac{0.5}{0.6} = \frac{0.6}{0.6} \cdot P(Q|H)$
$5/6 = P(Q|H)$

Ex. Action + Sci-Fy
A = Action
$P(A) = 0.7$

S = SciFy
$P(S) = 0.6 \quad P(A|S) = 0.65$

Find $P(S|A) = 39/70$

$P(A \text{ and } S) = P(S) \cdot P(A|S)$
$.39 \quad 0.6 \cdot 0.65$
$P(A \text{ and } S) = P(A) \cdot P(S|A)$
$\frac{.39}{.07} = \frac{0.7}{0.7} \cdot P(S|A)$
$= 39/70$

Ex. Bread (2) white, rye | Meat (2) turkey, roast beef | cheese (3) Provolone, cheddar, muenster

$2 \cdot 2 \cdot 3 = 12$ total sandwiches

A = roast beef
$P(A) = 6/12 = 1/2$

B = rye + muenster
$P(B) = 2/12 = 1/6$

$P(A|B) = 1/2$
$P(B|A) = 1/6$

Ex. S = Soccer
$P(S) = 0.4$

M = movies
$P(M) = 0.3$ $P(M|S) = 0.1$

$P(S \text{ and } M) = P(S) \cdot P(M|S)$
$0.04 = .4 \cdot .1$

Ex. M = Mystery
$P(M) = 0.6$

D = Drama
$P(D) = 0.8$ $P(D|M) = 0.7$

$P(D \text{ and } M) = P(M) \cdot P(D|M)$
$.42 = 0.6 \cdot 0.7$

Ex. C = Cereal
$P(C) = 0.4$

S = Sandwich
$P(S) = 0.6$ $P(S|C) = 0.7$

find $P(C|S)$
$7/15$

$P(C \text{ AND } S) = P(C) \cdot P(S|C)$
$.28 = .4 \cdot .7$

$P(C \text{ AND } S) = P(S) \cdot P(C|S)$
$\frac{.28}{.0.6} = 0.6 \cdot x$

Ex. 5 red marbles, 3 blue marbles

$P(A) = 5/8$

$P(B)$ = red → blue, blue → blue

$5 \cdot 3$, $3 \cdot 2$

$15 + 6 = \frac{21}{56} = \frac{3}{8}$

$3 \cdot 7 = 56$

$P(A \text{ and } B) = P(A) \cdot P(B)$

$15/64 = 5/8 \cdot 3/8$

Ex. 10 pens, 7 blue, 3 green

A = Blue first

$P(A) = 7/10$

B = green second

$P(B)$ = blue → green, green → green

$7 \cdot 3$, $3 \cdot 2$

$21 + 6 = \frac{27}{90} = \frac{3}{10}$

$10 \cdot 9 = 90$

$P(A \text{ and } B) = P(B) \cdot P(A|B)$

$7/30 \quad 3/10 \cdot 7/9$

Ex. Pair 6-sided dice

A = First die is 1

$P(A) = 1/6$

B = sum is 3

$P(B) = 1/18$

$P(A|B) = 1/2$

$P(B|A) = 1/6$

$P(A \text{ and } B) = 1/36$

$P(A) \cdot P(B|A)$

$1/6 \cdot 1/6 = 1/36$

$P(B) \cdot P(A|B)$

$1/18 \cdot 1/2 = 1/36$

Ex. Pair 6-sided

A = First is 2 or greater

$P(A) = 5/6$

B = Second is a 1

$P(B) = 1/6$

$P(A) \cdot P(B|A)$

$5/6 \cdot 1/6 = 5/36$

$P(B) \cdot P(A|B)$

$1/6 \cdot 5/6 = 5/36$

Ex. T = treadmill B = bench press

$P(T) = 0.5$ $P(B) = 0.9$ $P(B|T) = 0.8$

$P(A \text{ and } B) = P(T) \cdot P(B|T)$

$0.4 = 0.5 \cdot 0.8$

$P(A \text{ and } B) = P(B) \cdot P(T|B)$

$\frac{0.4}{0.9} \quad \frac{0.9}{0.9} \quad x$

$x = 4/9$

Ex. Flip fair coin twice

A = 2 tails B = ≤ one head (can be ∅ heads)

$P(A) = 1/4$ $P(B) = 3/4$

$P(A) \cdot P(B|A)$ $P(B) \cdot P(A|B)$

$1/4 \cdot 1 = 1/4$ $3/4 \cdot 1/3 = 1/4$

- Identifying Dependent and Independent Events

Ex. Playing Cards
$9\heartsuit_{B}, 5\spadesuit_{AB}, 6\heartsuit, 2\spadesuit_{A}, 4\heartsuit, 7\heartsuit_{B}$

A = ♠ (spade) $P(A) = 2/6 = 1/3$

B = odd # $P(B) = 3/6 = 1/2$

- $P(A|B) = P(A)$
  $1/3 = 1/3$ ✓

- $P(B|A) = P(B)$
  $1/2 = 1/2$ ✓

- $P(A \text{ and } B) = P(A) \cdot P(B)$ = Independent ✓
  $1/6 = 1/3 \cdot 1/2$
  $1/6 = 1/6$ ✓

Ex. Pair 6-sided Die
A = 1st die is 4 $P(A) = 6/36 = 1/6$

B = doubles $P(B) = 6/36 = 1/6$

$P(A \text{ and } B) = P(A) \cdot P(B)$ = Independent ✓
$1/36 = 1/6 \cdot 1/6$
$1/36 = 1/36$ ✓

Ex. 1 red shirt, 1 green shirt, 1 red hat, 1 green scarf,
1 pr. red pants, 1 pr. green pants

A = Green
$P(A) = 3/6 = 1/2$

B = scarf
$P(B) = 1/6$

- $P(A|B) = P(A)$
  $1 = 1/2$ ×

- $P(B|A) = P(B)$
  $1/3 = 1/6$ ×

- $P(A \text{ and } B) = P(A) \cdot P(B)$
  $1/6 = 1/2 \cdot 1/6$
  $1/6 = 1/12$ ×

Ex. 300 students; 120 sister, 100 brother, 40 both S+B
A = Sister
$P(A) = 120/300 = 2/5$

B = brother
$P(B) = 100/300 = 1/3$

$P(A|B) = 40/100 = 2/5$

- $P(A|B) = P(A)$
  $2/5 = 2/5$ ✓

- $P(A \text{ and } B) = P(A) \cdot P(B)$
  $40/300 = 2/5 \cdot 1/3$
  $2/15 = 2/15$ ✓ ∴ Independent

Ex. Chloe's Books: 10 books; 3 blue, 7 red

Blue 2 of blue are: mysterys, 1 fantasy

Red 5 of red are: mysterys, 2 fantasy

A = red book
$P(A) = 7/10$

B = mystery
$P(B) = 7/10$

- $P(A|B) = P(A)$
  $5/7 = 7/10$ ×

- $P(B|A) = P(B)$
  $5/7 = 7/10$ ×

- $P(A \text{ and } B) = P(A) \cdot P(B)$
  $5/10 = 7/10 \cdot 7/10$
  $1/2 = 49/100$ ×

Independent Ex. Deck of cards

A = card is a 2
$P(A) = 4/52 = 1/13$

B = Spade
$P(B) = 13/52 = 1/4$

- $P(A|B) = P(A)$
  $1/13 = 1/13$ ✓

- $P(A \text{ and } B) = P(A) \cdot P(B)$
  $1/52 = 1/13 \cdot 1/4$
  $1/52 = 1/52$ ✓ ∴ Independent

- $P(B|A) = P(B)$
  $1/4 = 1/4$ ✓

-Multiplying Dependent Probabilities

Always True

① $P(A \text{ and } B) = P(A) \cdot P(B|A)$

② $P(A \text{ and } B) = P(B) \cdot P(A|B)$

Ex. BBB BGB GGB BGG
BBG GBB GBG GGG

A = 1st child boy
$P(A) = 4/8 = 1/2$

B = All three boys
$P(B) = 1/8$

- $P(A|B) = 1$
- $P(B|A) = 1/4$
- $P(A \text{ and } B) = 1/8$
- $P(A) \cdot P(B|A)$
  $1/2 \cdot 1/4 = 1/8$
- $P(B) \cdot P(A|B)$
  $1/8 \cdot 1 = 1/8$

Ex. Flip fair coin twice

A = 1st flip heads $P(A) = 2/4 = 1/2$

B = 2nd flip tails $P(B) = 2/4 = 1/2$

H H
H T
T H
T T

$P(A \text{ and } B) = 1/4$

- $P(A \text{ and } B) = P(A) \cdot P(B)$
  $1/4 = 1/2 \cdot 1/2$
  $1/4 = 1/4$ ✓ ∴ Ind.

- $P(A \text{ and } B) = P(A) \cdot P(B|A)$
  $1/4 = 1/2 \cdot 1/2$
  $1/4 = 1/4$ ✓

- $P(A \text{ and } B) = P(B) \cdot P(A|B)$
  $1/4 = 1/2 \cdot 1/2$ ✓

Ex. Girl @ gym

Y = yoga

S = swims
$P(S) = 0.7$

$P(S|Y) = 0.6$
$P(Y|S) = 0.5$

$P(S \text{ and } Y) = P(S) \cdot P(Y|S)$
$.35 = 0.7 \cdot 0.5$

Find: $P(Y)$

$P(S \text{ and } Y) = P(Y) \cdot P(S|Y)$
$\frac{.35}{0.6} = \frac{x \cdot 0.6}{0.6}$
$x = .583$

Using Probability to Make Fair Decisions

① P: RB $\frac{26}{52}\cdot\frac{25}{51}=.245$ A: BB $\frac{26}{52}\cdot\frac{25}{51}=.245$ RB $\frac{26}{52}\cdot\frac{26}{51}=.254$ BR $\frac{26}{52}\cdot\frac{26}{51}=.254$

② (1,1) (2,1) 3,1 4,1 (5,1) (6,1)
1,2 (2,2) (3,2) (4,2) (5,2) (6,2)
1,3 2,3 (3,3) (4,3) 5,3 6,3
1,4 (2,4) (3,4) 4,4 (5,4) 6,4
(1,5) (2,5) 3,5 4,5 5,5 (6,5)
(1,6) (2,6) 3,6 4,6 5,6 (6,6)

	A	B	C	
①	6	6	6	36 − 6 = 30
②	6	15	15	

③ $\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}=\frac{1}{216}$ $\frac{1}{6}\cdot\frac{5}{6}\cdot\frac{4}{6}=\frac{20}{216}$ HH HT TH TT
N N C S

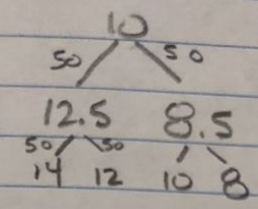
④ 1,4,9

Types of Statistical Studies

- Causality -vs- Correlation/Association

- Types of Studies
  1. Sample - estimates a certain parameter
  2. Experiment - need a "control" group
  3. Observational - Look @ "all"

Constructing Probability Distributions



HJ	TJ	$\frac{2}{8} = \frac{1}{4}$
HQ	TQ	
HK	TK	
HA	TA	

$f(n) = x^n$

① $2/5 = 3/5$ ×

② $2/5 \cdot 7/10 \quad 14/50 = 7/25$ ×

③

n	1	2
m	$\frac{7}{10}$	
S	$\frac{11}{30}$	

④

1	2
$\frac{2}{5}$	$\frac{3}{5}$

mm	$\frac{1}{2/5} \cdot \frac{2}{7/10} = 14/50 = 7/25 = .28$
ms	$2/5 \cdot 3/10 = 6/50 = 3/25 = .12$
sm	$3/5 \cdot 11/30 = 33/150 = 11/50 = .22$
ss	$3/5 \cdot 19/30 = 19/50 = .38$